Angular momentum loss from cool stars: an empirical expression and connection to stellar activity

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ABSTRACT

We show here that the rotation period data in open clusters allow the empirical determination of an expression for the rate of loss of angular momentum from cool stars on the main sequence. One significant component of the expression, the dependence on rotation rate, persists from prior work; others do not. The expression has a bifurcation, as before, that corresponds to an observed bifurcation in the rotation periods of coeval open cluster stars. The dual dependencies of this loss rate on stellar mass are captured by two functions, $f(B-V)$ and $T(B-V)$, that can be determined from the rotation period observations. Equivalent masses and other $[UBVRIJHK]$ colors are provided in Table 1. Dimensional considerations, and a comparison with appropriate calculated quantities suggest interpretations for f and T, both of which appear to be related closely (but differently) to the calculated convective turnover timescale, τ_c, in cool stars. This identification enables us to write down symmetrical expressions for the angular momentum loss rate and the deceleration of cool stars, and also to revive the convective turnover timescale as a vital connection between stellar rotation and stellar activity physics.

Subject headings: convection - open clusters and associations: general - stars: activity - stars: evolution - stars: late-type - stars: rotation

1. Introduction

Cool stars are known to lose angular momentum and spin down over time. However, a detailed understanding of this loss has yet to be achieved. Measurements from the ground and space, coupled with theory, give us a good idea of the rate of angular momentum loss for the present-day Sun, but extensions to stars of different masses or those of other ages, particularly very young stars, are problematical in various ways. This paper proposes a path toward making such extensions.

In the same paper that proposed the existence of the solar wind, Parker (1958) noted that this wind would cause a 'retardation of solar rotation' on a timescale similar to its lifetime. Weber & Davis (1967) elaborated on this particular effect and showed that the associated angular momentum loss rate is equivalent

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to assuming corotation of the solar wind out from the surface to the Alfvenic radius, r_A. Schatzman (1962) had previously suggested that the solar case could be generalized to other stars, linked the associated angular momentum loss to the presence of a surface convection zone, and provided a formula for the loss rate in terms of a star's angular velocity and certain other quantities. However, it was not clear how to calculate or measure these quantities. A similar criticism could be leveled against the expression for angular momentum loss provided by Mestel (1968). The associated viewpoint that the angular momentum loss rate, $dJ/dt \propto \Omega r_A^2 \dot{M}$ where Ω and $\dot{M}$ are the star's angular velocity and mass loss rate respectively, ultimately runs into the difficulty of providing credible calculations or measurements of both r_A and $\dot{M}$ for the Sun and cool stars of other masses. Alternatively, writing $dJ/dt \propto B_0^2 R^4 \Omega / v_A$, where B_0, R, and v_A are the surface magnetic field, stellar radius, and wind velocity at the Alfven surface, requires deriving or measuring B_0 and v_A as a function of a star's mass and other properties. See Collier Cameron & Li (1994) for a discussion of the issues that arise when one takes this viewpoint.

Another way to approach this problem is empirical. Kraft (1967) measured a decline of stellar rotation velocities with age, strengthening the solar-stellar analogy, and Schatzman's work provided a natural framework for interpreting the break in the Kraft (1967) curve of rotation velocities against stellar mass. The work of Skumanich (1972) provided more direct information about the angular momentum loss rates for solar-mass stars when he noted that their measured rotation velocities, $v \sin i$, decline with age, t, as

$$\overline{v \sin i} \propto t^{-0.5} \tag{1}$$

where $\overline{v \sin i}$ represents an average over coeval cluster stars. This observation implies that the rate of loss of angular momentum for (solar-mass) stars of constant structure (i.e. moment of inertia) obeys the relationship

$$\frac{dJ}{dt} \propto -\Omega^3. \tag{2}$$

According to this viewpoint, if the observations can provide the acceleration, $d\Omega/dt$ as a function of all relevant variables, then dJ/dt immediately follows if the associated moment of inertia can be inferred.

A hybrid approach helps overcome certain weaknesses of both viewpoints. Endal & Sofia (1981) used the wind-loss formulation of Belcher & MacGregor (1976), in turn based on the work of Weber & Davis (1967), to understand the rotational history of the Sun, but the resulting rotational evolution (their Figure 4), including certain assumptions about internal transport of angular momentum, did not closely match the empirical results of Skumanich (1972). At that time, the security of the Skumanich result was not assured, and its applicability was restricted to stars of near-solar mass. Indeed, Mestel (1984) investigated other loss relationships than $dJ/dt \propto -\Omega^3$ and made significant steps toward elucidating what they implied. Kawaler (1988) extended Mestel's work in certain significant ways, identifying dependencies in addition to Ω in the expression for angular momentum loss, and proposed an expression that could be implemented easily in models of rotating cool stars of *varying* mass. The relationship, combining theoretical and observational considerations, is

$$\frac{dJ}{dt} = -K_w \Omega^{1+4an/3} \left(\frac{R}{R_\odot}\right)^{2-n} (\dot{M}_{14})^{1-2n/3} \left(\frac{M}{M_\odot}\right)^{-n/3} \tag{3}$$

where R, M, and $\dot{M}$ are the stellar radius, mass, and mass loss rate respectively, n parameterizes the geometry of the magnetic field, a is the exponent that relates the surface magnetic field, B_0 to Ω via $B_0 \propto \Omega^a$, and K_w is a calibration constant chosen to ensure that a solar model attains the solar rotation rate at solar age. n can vary from 3/7 for a dipolar field to 2 for a purely radial field. If one assumes a linear dynamo, then $a = 1$, and n must then be set equal to 1.5 to reproduce the observed Skumanich spin-down, $\overline{v \sin i} \propto t^{-0.5}$.

This choice of n kills the mass loss term, of course. Consequently, the above expression usually simplifies to

$$\frac{dJ}{dt} = -K_w \Omega^3 (\frac{R}{R_\odot})^{0.5} (\frac{M}{M_\odot})^{-0.5}, \tag{4}$$

eliminating the mass loss rate, $\dot{M}$. This expression, a hybrid of the two viewpoints discussed above, has been routinely used to drain angular momentum from rotational stellar models constructed using YREC, the Yale Rotating Stellar Evolution Code (e.g., Pinsonneault et al. 1989), and also is used in other stellar models (e.g., Bouvier et al. 1997). A didactic account of the above developments, set in a broader context than here, may be found in Chapters 13 and 21 of Maeder (2009).

We note that some studies of the rotational evolution of stars do not explicitly provide an expression for angular momentum loss (e.g., MacGregor & Brenner 1991; Armitage & Clarke 1996). This is usual in cases where the magnetic field and associated quantities (and sometimes their evolution) are themselves modeled numerically. The mass loss rate is usually an input parameter. A recent example is the work of Matt & Pudritz (2008), which provides details of such an approach. Some models include the possible effects of disks on the pre-main-sequence (e.g., Collier Cameron & Campbell 1993, Keppens et al. 1995, Sills et al. 2000, Barnes et al. 2001). But here we are concerned only with the main-sequence where any possible effect of pre-main-sequence disks has abated. We also note that cataclysmic variable (CV) research uses an expression derived by Rappaport et al. (1983) for the extra-gravitational loss of angular momentum. This expression has some similarity to the Kawaler (1988) expression, as noted by Andronov et al. (2003). However, CVs are beyond the scope of this paper.

In the observational domain, rotational data for stars in open clusters had revealed (and further observations confirmed) the presence of the so-called ultra-fast rotators (UFRs) in the Pleiades (van Leeuwen & Alphenaar 1982). These observations implied that the angular momentum loss rate for such stars was *much lower* than that calculated using Kawaler's expression. Chaboyer et al. (1995), following MacGregor & Brenner (1991), therefore suggested modifying this relationship at high rotation rates to facilitate the theoretical modeling of such stars. This idea became known as "saturation" (e.g., Stauffer 1994) and was roughly parallel to a similar phenomenon in soft X-rays, (although the exact connection between the two was not clear then). Chaboyer et al. (1995) suggested the loss relationship

$$\frac{dJ}{dt} = -K_w (\frac{R}{R_\odot})^{0.5} (\frac{M}{M_\odot})^{-0.5} \begin{cases} \Omega^3 & \text{for } \Omega \le \Omega_{\text{crit}}, \\ \Omega_{\text{crit}}^2\, \Omega & \text{for } \Omega > \Omega_{\text{crit}}, \end{cases} \tag{5}$$

where Ω_{crit} is a constant. (A way of achieving a similar effect in the context of Weber-Davis type wind models is presented in Collier Cameron & Li 1994.) Furthermore, Barnes & Sofia (1996) showed that the UFRs could not be modeled by the original Kawaler expression, with its Ω^3 dependence, regardless of any possible pre-main-sequence spin-up, the bloated state of T Tauri stars being considered a possible reservoir of angular momentum suited to explain the origin of the UFRs. As a result, subsequent work, e.g., Krishnamurthi et al. (1997) and Barnes et al. (2001) routinely used the newer Chaboyer et al. (1995) loss prescription. It was hoped that Ω_{crit} could simply be set equal to some constant threshold angular velocity for all relevant cool stars.

However, the accumulation of additional data suggested that a single constant value of Ω_{crit} for all cool stars was inadequate (Barnes & Sofia 1996; Krishnamurthi et al. 1997). Consequently, Krishnamurthi et al. (1997) advocated a scaling

$$\Omega_{\text{crit}} = \Omega_{\text{crit}\odot} \frac{\tau_\odot}{\tau_c} \tag{6}$$

where τ_c is the convective turnover timescale. (Prior to this, and in the context of using a Weber-Davis type wind to understand the rotational evolution of stars, Collier Cameron & Li (1994) had also suggested a scaling involving the convective turnover timescale, in this case setting the star's surface magnetic field, B_0 according to $B_0 \propto \tau_c \Omega$.)

The Ohio State University group (Sills et al. 2000; Andronov et al. 2003; Denissenkov et al. 2010) absorb the solar radius and mass from the Kawaler/Chaboyer formula into the wind constant, K_w, and write:

$$\frac{dJ}{dt} = -K_w \sqrt{\frac{R}{M}} \begin{cases} \Omega^3 & \text{for } \Omega \leq \Omega_{\text{crit}}, \\ \Omega_{\text{crit}}^2\, \Omega & \text{for } \Omega > \Omega_{\text{crit}}, \end{cases} \tag{7}$$

but Ω_{crit} is now variable as described immediately above, or varied piece-wise to match open cluster observations. For example, Sills et al. 2000 used the Krishnamurthi et al. (1997) scaling for Ω_{crit} for masses down to $0.6 M_\odot$, and set Ω_{crit} manually for each modeled stellar mass below this. Stars of very low mass are beyond the scope of this work, but we note that such a tuning effectively modifies Ω_{crit} from a constant into a new and arbitrarily modifiable function. Making Ω_{crit} mass dependent implies, of course, that the $\sqrt{R/M}$ term in the expression for dJ/dt is simply not capturing the mass dependence of the angular momentum loss, as it was originally intended to. Clearly, there is a problem.

On the other hand, the bifurcation in the angular momentum loss, as proposed by MacGregor & Brenner (1991), encapsulated in the relationship of Chaboyer et al. (1995), and as subsequently used in modeling is important in retrospect, because it does capture an essential feature of the observations. Indeed, since then, a steadily growing rotation period database has allowed the identification of distinct fast (C-) and slow (I) sequences in color-period diagrams of open cluster stars, as initially proposed by Barnes (2003). This C/I classification has been confirmed by extensive rotation period observations in M 35 (Meibom et al. 2009), M 37 (Hartman et al. 2009) and M 50 (Irwin et al. 2009), the first including a decade-long radial velocity survey for cluster membership and multiplicity. Scholz & Eisloffel (2007) include a discussion of this bifurcation vis-a-vis rotation periods in Praesepe/Hyades, and most recently, data in Hartman et al. (2010) clearly display this bifurcation in a large and uniform rotation period study of the Pleiades.

This observed bifurcation ties in well with the bifurcation in the angular momentum loss expression proposed by Chaboyer et al. (1995), although not to the mass dependence used there, which is the same for both kinds of stars. We shall see below that the mass dependence is actually different for the two sequences, so that it is *impossible* for the same expression to describe both.

Barnes (2003) also suggested interpretations of the observed shapes of the C- and I sequences, based on theoretical considerations, and a unifying scenario (hereafter called the CgI scenario) proposed in that work for the rotational evolution of cool stars. In particular, he suggested that stars initially are fast rotators on the C sequence, where the inner radiative and outer convection zones are largely decoupled, so that the mass dependence of this sequence is specified by (the reciprocal of) the moment of inertia of the outer convection zone. The observed transition of stars from the C- to the I sequence was proposed to be coincident with a change in the mass dependence, which was itself proposed to change from that of the outer convection zone alone to that of the entire star, and also to be coincident with the onset of an interface dynamo. Thus the mass dependence for the I sequence was suggested to be dependent on (the reciprocal of the square root of) the moment of inertia of the whole star.

This work begins by considering and checking whether these proposed dependencies work. We show that they do not (Section 2). We then show that the observations themselves might be queried to provide dJ/dt directly in terms of observed quantities (Section 3). Section 4 proposes an interpretation of the relevant

observed quantities in terms of the convective turnover timescale. The relationship with stellar activity physics is pointed out in Section 5, and the conclusions are stated immediately thereafter. (The next paper in this series combines the C- and I-type behaviors identified in Section 4 into a simple nonlinear model for the rotational evolution of stars and explores the consequences for gyrochronology.)

2. Inadequacy of the moment of inertia proposal

We begin by negating a proposal made by Barnes (2003), that the mass dependence of the rotation periods in open clusters can be simply attributed to the moments of inertia of either the whole star or that of the surface convection zone.

2.1. I sequence

Kawaler (1989) was the first to note that beyond an age of a few hundred million years, as exemplified by the 600 Myr-old Hyades open cluster, a deterministic relationship between rotation period, color and age could be derived, and inverted to provide a star's age. Younger clusters have a more complicated morphology, initially parsed into fast/C- and slow/I sequence stars by Barnes (2003). He proposed that the latter, I sequence stars are describable by $P_I = f(B - V) \times g(t)$ where $f(B - V)$ and $g(t)$ are empirically determinable functions of the $B - V$ color and age, t, respectively. In that paper, the functional forms used were $f(B-V) = \sqrt{(B - V - 0.5)} - 0.15(B - V - 0.5)$ and $g(t) = \sqrt{t}$. These functions have been subsequently re-determined, most notably by Meibom et al. (2009), based on a very large study of both rotation periods and membership in the open cluster M 35. They determined that $f(B - V) = 0.77(B - V - 0.47)^{0.55}$, and for definiteness, we will use this latter form in this paper[1]. (Transformation to mass or other colors in the set $[UBVRIJHK]$ can be accomplished using Table 1.)

Barnes (2003) suggested identifying $f(B-V)$ with $1/\sqrt{I_*}$, where I_* is the moment of inertia of the star. We have calculated I_* using the latest version of YREC, and display it in Figure 1 as a function of mass and of $B - V$ color for a series of 500 Myr models of solar composition. This age was selected to ensure that all lower-than-solar-mass stellar models of interest have passed through the pre-main-sequence phase and arrived on the main-sequence, while higher-than-solar-mass models have not evolved off the main-sequence. To the precision of this work, further main-sequence evolution does not have an appreciable effect. The numerical values and associated $[UBVRIJHK]$ colors using both Green et al. (1987) and Lejeune et al. (1997), Lejeune et al. (1998) color transformations are provided in Table 1.

We compare $f(B - V)$ with $1/\sqrt{I_*}$ in Figure 2. The curves are normalized to agree for the solar case. It is clear that the agreement is not good. One could argue that the normalization makes it hard to tell how poor the fit is for the cooler stars. The key part of the disagreement, however, is that $1/\sqrt{I_*}$ does not drop off sufficiently fast for the warmer stars. This disagreement would only get worse if the normalization were changed.

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[1] In a later comparison, we will also show the Mamajek & Hillenbrand (2008) determination of $f(B-V)$. We also acknowledge that the index n in $g(t) = t^n$ is usually found to exceed 0.5 slightly (e.g., Barnes 2007, Collier Cameron et al. 2009, James et al. 2010).
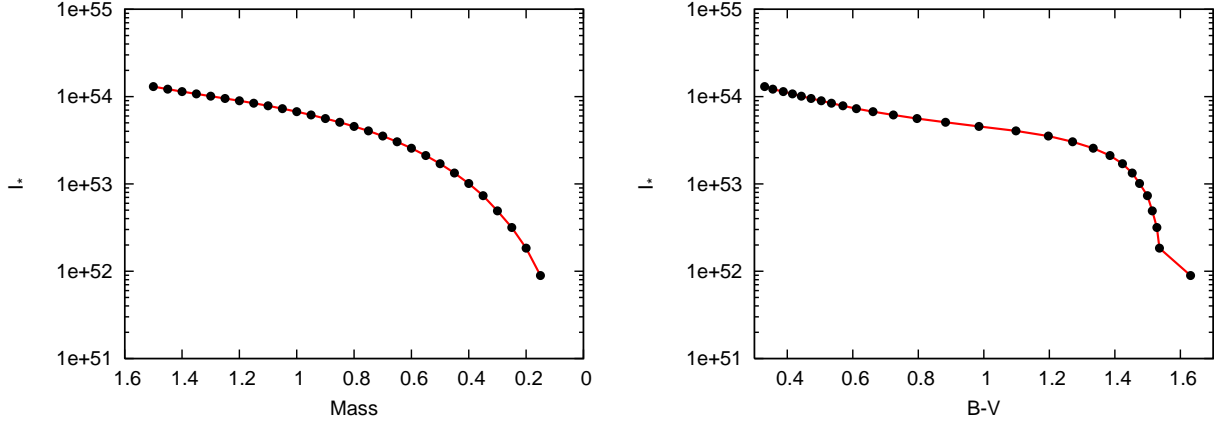
Fig. 1.— **Left:** The moment of inertia of the entire star, I_*, against stellar mass, in the range $1.5 - 0.15 M_\odot$. (Note the inverted mass axis.) **Right:** The same against $B - V$ color. As expected, I_* increases monotonically with stellar mass. (In this, and subsequent figures, we display the Lejeune et al. (1997, 1998) $B - V$ color; other colors in the set $[UBVRIJHK]$, including those with the Green et al. (1987) color transformations are provided in Table 1.)

2.2. C sequence

Barnes (2003) proposed that the C sequence of faster rotating stars can be described by $P_C = P_0 e^{t/T(B-V)}$, where P_C denotes the rotation periods of the stars in question, t is the age, and $T(B - V)$ is the relevant spin-down timescale[2]. The open cluster observations indicate that $T(B-V)$ is short for stars bluer than the Sun, tending toward zero at mid-F. On the other side, redward of the Sun, $T(B - V)$ is known to increase. The main requirement is a function that dives to zero at $B - V = 0.47$, the x-intercept of $f(B - V)$, as determined by Meibom et al. (2009). The most reliable empirical determinations of $T(B - V)$ for G and K stars are also by Meibom et al. (2009), who find that $T(B - V) = 60\,\mathrm{Myr}$ for G stars (specifically, those with $0.6 < B - V < 0.8$) and $T(B - V) = 140\,\mathrm{Myr}$ for K stars, (with $0.8 < B - V < 1.3$). We estimate uncertainties on these values of 5 Myr and 13 Myr respectively, and use these values to represent those for mean $B - V$ colors of 0.7 and 1.05. For redder stars, we note that about a third of the Hartman et al. (2009) sample of stars from the 550 Myr-old open cluster M37 are on the C sequence. This implies that $T(B - V) = 500 \pm 65\,\mathrm{Myr}$ for early M stars, represented by a point at $B - V = 1.45$. $T(B - V)$ is plotted using these discrete values in Figure 3. We also display a cubic spline fit to these data, and acknowledge that $T(B - V)$ becomes increasingly uncertain with decreasing stellar mass[3]. Similar results - $50 - 100\,\mathrm{Myr}$ for $1 M_\odot$, $0.5 - 1\,\mathrm{Gyr}$ for $0.5 M_\odot$, and several Gyr for $M < 0.3 M_\odot$ - were quoted by Scholz et al. (2009) and references therein.

Barnes (2003) suggested that $T(B-V) \propto I_{cz}$, where I_{cz} is the moment of inertia of the convection zone. We have calculated I_{cz} using YREC, and display it in Figure 4 as a function of mass and of $B - V$ color for a series of 500 Myr models of solar composition. I_{cz} drops at both ends as expected; it decreases for stars with masses lower than $0.8\,M_\odot$, and also for the warmer stars with thinning surface convection zones. The

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[2]The choice of an exponential function in t is partly motivated by theoretical considerations, and it is possible that other functions of t are also suitable, as discussed below. $T(B - V)$ is our main concern here.

[3]The uncertainties quoted above are based on fractional numbers of stars, and do not include other effects such as uncertainties in the assumed cluster isochrone ages.
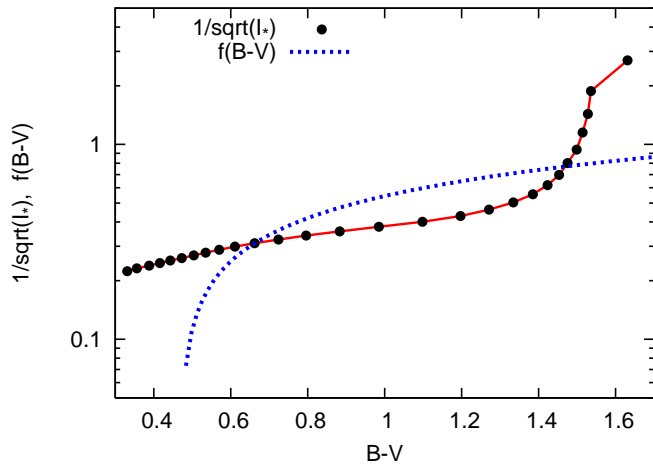
Fig. 2.— Comparison of the empirical function $f(B-V)$ for the I sequence (dashed blue), with $1/\sqrt{I_*}$ (filled circles). The latter was proposed by Barnes (2003) to interpret the empirical dependence. (The I_* curve is normalized for solar mass.) We conclude that the two curves are not as similar as was proposed in Barnes (2003), negating the corresponding identification. In particular, $1/\sqrt{I_*}$ does not drop steeply enough to model the observed rotation period decline among the warmer stars.

associated numerical values and other colors are provided in Table 1.

We compare $T(B-V)$ with I_{cz} in Figure 5. The two variables are in reasonable agreement for $M > M_\odot$ but diverge badly for $M < M_\odot$. Whereas $T(B-V)$ is a steadily increasing function of $B-V$ (or decreasing stellar mass), I_{cz} becomes relatively flat for K dwarfs, and indeed drops for M dwarfs, as might be expected. Consequently, we find that I_{cz} is a poor match to $T(B-V)$, and withdraw the corresponding suggestion made in Barnes (2003).

3. Empirical approach

It transpires that the observations themselves can be queried to provide an empirical loss rate, through a logical extension of prior work. This is accomplished here, considering first the I type rotators, and then the C type ones.

3.1. I sequence

We return to the observational basis that the rotation periods, P_I, of stars on the I sequence are describable as a product of two separable functions, f, and g, of the star's mass or $B - V$ color and age, t, respectively:

$$P_I = f(B - V) \times g(t). \tag{8}$$

This was shown by construction in Barnes (2003) and Barnes (2007), where both $f(B - V)$ and $g(t)$ were determined empirically.
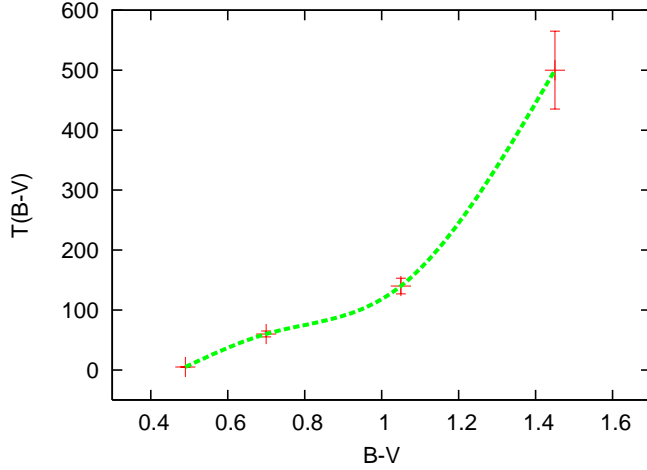
$$g(t) = \sqrt{t} \tag{9}$$

Fig. 3.— The C sequence spin-down timescale, $T(B - V)$, for cool stars (in units of Myr) plotted against $B - V$ color. The green line is a cubic spline fit to the data. $T(B - V)$ increases steadily with $B - V$ color, somewhat faster than linear for the coolest stars. The data points are discussed in the text.

was found there to be applicable to a wide range of cool stars, a result tracing back to the work of Skumanich (1972), where the more restrictive case of solar-mass stars was considered. As in the prior section, we will use the Meibom et al. (2006) determination

$$f(B - V) = 0.77(B - V - 0.47)^{0.55} \tag{10}$$

over other determinations, including those of Barnes (2003) and Barnes (2007), because of the size and quality of the data set considered there.

Changing variable from P_I to Ω via $P_I = 2\pi/\Omega$, equation (8) becomes

$$\frac{2\pi}{\Omega} = f(B - V) \times g(t), \tag{11}$$

and upon differentiating it with respect to time, we get

$$\frac{d\Omega}{dt} = -\frac{\Omega^2}{2\pi} f(B - V) \frac{dg}{dt}. \tag{12}$$

Using $g(t) = \sqrt{t}$ from Equation (9), we get

$$\frac{d\Omega}{dt} = -\frac{\Omega^2}{4\pi} f(B - V) \frac{1}{\sqrt{t}}. \tag{13}$$

Re-using Equation (11) to write $\sqrt{t}$ in terms of f and Ω, we get

$$\frac{d\Omega}{dt} = -\frac{\Omega^3}{8\pi^2} f^2(B - V), \tag{14}$$

which specifies the deceleration of the star in purely observational quantities. Note that (the square of) $f(B - V)$ completely specifies the mass dependence. $f(B - V)$ being zero for mid-F and earlier-type stars, the deceleration is correspondingly zero for these stars. $f(B - V)$ increases steadily through late-F, G, K,
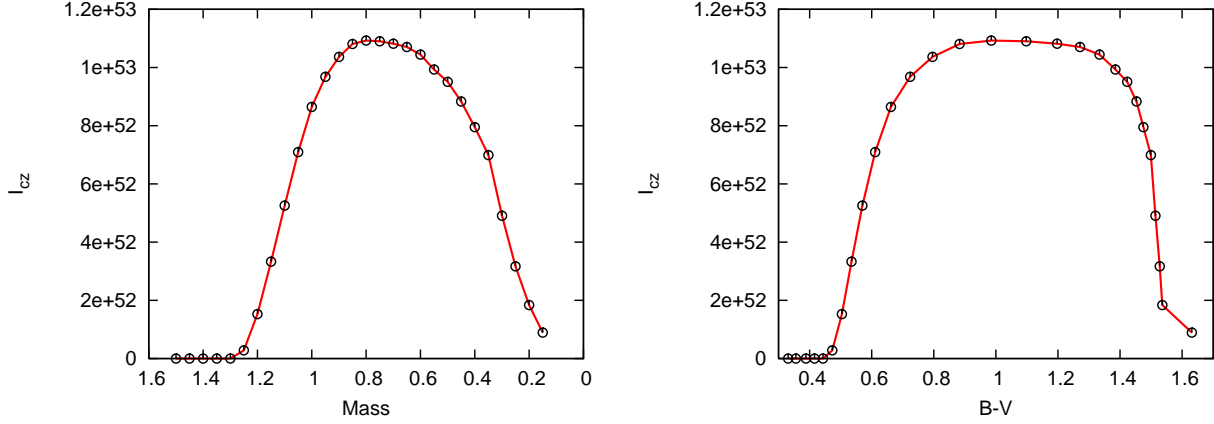
Fig. 4.— **Left:** The moment of inertia of the outer convection zone, I_{cz}, against stellar mass. (Note the inverted mass axis.) **Right:** The same against $B-V$ color. I_{cz} drops at both ends as expected; to the left as surface convection zones thin out and vanish, and to the right, with diminishing stellar mass.

and early M stars, so that the deceleration of these stars is correspondingly greater. This accounts for the shape of the I sequence in open clusters. It is also worth emphasizing that the open cluster data directly provide only the deceleration above, not the angular momentum loss rate, dJ/dt, derived below.

Deriving the rate of angular momentum loss requires an additional step. By the Chain Rule,

$$\frac{dJ}{dt} = I\frac{d\Omega}{dt} + \Omega\frac{dI}{dt} \tag{15}$$

so that simple substitution from Equation (14) above gives

$$\frac{dJ}{dt} = -\frac{\Omega^3}{8\pi^2}If^2(B-V) + \Omega\frac{dI}{dt}. \tag{16}$$

Barnes (2003) suggested, in agreement with results from helioseismology, that the relevant moment of inertia I, in this case is I_*, the moment of inertia of the entire star. To a very good approximation, this is constant on the main-sequence, killing the final term in Equation (16), so that the final empirical expression for the rate of loss of angular momentum, dJ/dt, from I sequence stars becomes

$$\frac{dJ}{dt} = -\frac{\Omega^3}{8\pi^2}I_*f^2(B-V). \tag{17}$$

This expression has the appealing feature that $f(B-V)$ can be straight-forwardly determined from open cluster rotation period data, provided that the memory of the possibly complex initial conditions has been erased. We know this to be at least approximately true on the I sequence, because the Sun is on this sequence, and because helioseismic results indicate that the Sun is a solid-body rotator to first order. Consequently, expression (17) combines a relatively well-known and easily calculated function I_*, with the observationally well-determined function $f(B-V)$ to state the mass dependence of angular momentum loss, *independent of assumptions beyond those encapsulated in Equations (8) and (9) above.* On the negative side, it has the undesirable feature of mixing calculated and observed quantities in one expression. It is also not explanatory, because the origin of $f(B-V)$ is not yet specified. In a section below, we propose an identification for $f(B-V)$.
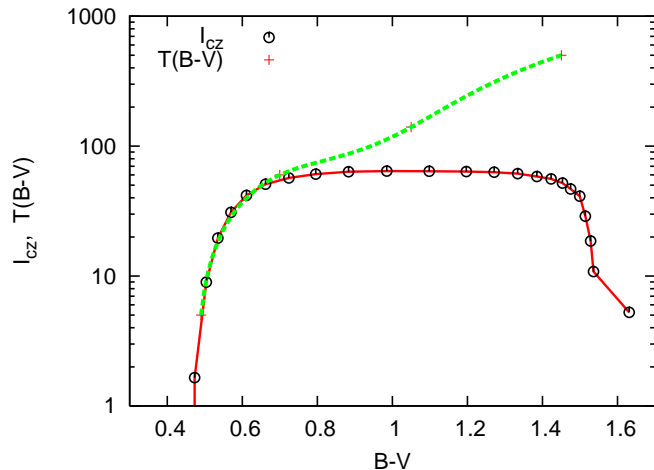
Fig. 5.— The C sequence spin-down timescale $T(B - V)$, in units of Myr (pluses; dashed green line), is compared with I_{cz}, the moment of inertia of the surface convection zone in cool stars (unfilled circles; solid red line). I_{cz} is normalized for solar mass. Although the agreement is good for $M > M_\odot$, partly because both I_{cz} and T drop to zero at similar color, I_{cz} and $T(B - V)$ can be observed to diverge for lower mass stars, negating the corresponding identification proposed in Barnes (2003).

We note that assuming $f \propto 1/\sqrt{I_*}$, as suggested in Barnes (2003), would simply replace $I_* f^2$ with a constant, and would not provide the correct mass dependence observed in the rotation period data, as shown above. Consequently, we here withdraw that suggestion. Also, expression (17) above can be compared directly with the $\Omega \leq \Omega_{\rm crit}$ part of the prior loss rate expression (7) which is:

$$\frac{dJ}{dt} = -K_w \sqrt{\frac{R}{M}} \Omega^3 \tag{18}$$

Comparing this with Equation (17) above shows that, apart from numerical constants, the difference in the loss relationships boils down to the difference between $\sqrt{R/M}$ and $I_* f^2$.

We plot both $I_* f^2$ and $\sqrt{R/M}$ against $B - V$ color in Figure 6, so that a graphical comparison can be made. As before, the curves are normalized for a solar-mass model. Redward of $B - V = 1.4$, for very low mass stars, the physics is expected to be different, and neither relationship might be relevant. So, only the area blueward of this is relevant. Unsurprisingly, R/M, and consequently, $\sqrt{R/M}$, is a relatively flat function of $B - V$ color for FGK stars, because the stellar radius tracks the mass. Consequently, the angular momentum loss rate for the prior prescription is essentially independent of spectral type, and *any assumed initial distribution of rotation periods can be expected to retain this initial shape when subjected to this loss rate.* Consequently, with the prior loss prescription, the observed shape of $f(B - V)$ would have to be assumed as an initial condition. It would not arise as a natural consequence of the angular momentum loss prescription.

In contrast, $I_* f^2$ is only somewhat flat for K stars ($B - V \sim 1$), where it attains its maximum (Figure 6), and declines sharply blueward for G stars, dropping to zero blueward of late-F. This mass-dependence has been derived from the data, so it must be true within the uncertainties of the data themselves and the above assumptions. Consequently the loss rate is maximized at K stars, but declines steadily for G stars, dropping to zero at mid-F. Here, *almost any non-pathological set of initial periods would progressively attain the shape of $f(B - V)$ with the passage of time.*
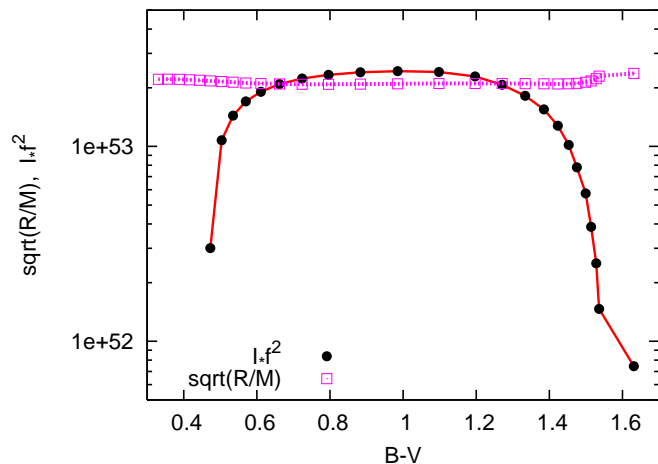
Fig. 6.— Comparison of the empirical dependence, $I_* f^2$ (filled circles), proposed here for the I sequence, with the prior suggested color dependence of angular momentum loss, $\sqrt{(R/M)}$, (unfilled squares). The curves are normalized for the solar case. $\sqrt{R/M}$ is observed to be significantly different from the empirically determined function $I_* f^2$.

3.2. C sequence

Here, we derive the second half of the angular momentum loss expression, for the C sequence, in a manner symmetrical to that for the I sequence above. We can tackle the C sequence in a manner similar to the I sequence, although without recourse to separability of the mass and age dependencies, by writing the rotation periods, P_C, of the C sequence stars as a function

$$P_C = P_C(B - V, t) \tag{19}$$

of the $B - V$ color and age, t, of the star. The open cluster observations show that the rotation periods of the C sequence stars have a roughly exponential behavior with age[4], suggesting that we write the periods of these stars in a form originally suggested by Barnes (2003):

$$P_C = P_0 e^{t/T(B-V)} \tag{20}$$

where P_0 is the initial period for C sequence stars, and $T(B-V)$ is the appropriate mass-dependent timescale for spin-down on the C sequence, as discussed in Section 2.2 earlier. The open cluster data currently provide $T(B-V)$ only in discrete form. Figure 3 displays $T(B-V)$ as a function of $B - V$ color, along with a cubic spline fit.

Switching variable from P_C to Ω using $P_C = 2\pi/\Omega$, equation (20) becomes

$$\frac{2\pi}{\Omega} = P_0 e^{t/T(B-V)} \tag{21}$$

and on differentiating it with respect to time, we get

$$\frac{d\Omega}{dt} = -\frac{\Omega^2}{2\pi} P_0 \frac{e^{t/T(B-V)}}{T(B-V)}. \tag{22}$$

---

[4]An exponential is a compact way of expressing this dependence because it can be expanded in a power series in t/T and the coefficients determined. For example, for late-type stars with large T, the exponential will devolve into a function that is linear in t/T and will also lead to the same final result for $d\Omega/dt$.

Resubstituting from Equation (20) and again using $P_C = 2\pi/\Omega$, we get

$$\frac{d\Omega}{dt} = -\frac{\Omega}{T(B-V)}, \tag{23}$$

which again specifies the deceleration of the star in purely observational quantities, this time for C sequence stars. Note that the mass dependence of the deceleration is specified completely by $1/T(B-V)$. Given the form of $T(B-V)$ from Section 2.2, with $T(B-V)$ increasing monotonically with $B-V$ color, we see at once that as far as C sequence stars are concerned, the deceleration is greatest for late-F stars, and declines steadily as the mass decreases through G, K, and M stars. We note that the data provide only the deceleration above, not the angular momentum loss rate, dJ/dt, derived below. However, dJ/dt is required for comparison with prior work, and is potentially important in deriving conclusions about the mass loss rate and the Alfven Radius.

Deriving the angular momentum loss rate requires additional steps. Applying the Chain Rule as before,

$$\frac{dJ}{dt} = I_C\frac{d\Omega}{dt} + \Omega\frac{dI_C}{dt}, \tag{24}$$

so that substitution from above gives

$$\frac{dJ}{dt} = -\frac{\Omega}{T(B-V)}I_C + \Omega\frac{dI_C}{dt}. \tag{25}$$

We do not know yet what I_C, the relevant moment of inertia on the C sequence, is; but if we assume that it is not varying with time, we get as the final empirical expression for the rate of loss of angular momentum, dJ/dt, from C sequence stars

$$\frac{dJ}{dt} = -\Omega\frac{I_C}{T(B-V)}. \tag{26}$$

As with the I sequence case, $T(B-V)$ is derivable from the data, so that the rate of loss of angular momentum immediately follows provided that we can guess the appropriate moment of inertia I_C, for C sequence stars, and provided that this moment of inertia is not varying with time. (We will revisit this last assumption in a subsequent publication.) As with the I sequence, $T(B-V)$ is empirical, and thus not explanatory, because its origin has not been specified. In the next section, we propose an identification for $T(B-V)$.

There is little choice about (a time-independent) I_C. It could either be the moment of inertia of the entire star, I_*, or, if one followed the suggestion of Barnes (2003), the moment of inertia of only the outer convection zone, I_{cz}. If $I_C = I_*$, then dJ/dt would increase steeply with stellar mass, as shown in Figure 7 (left panel, filled circles), because the increase in I_* with stellar mass is amplified by the concomitant decrease in $T(B-V)$. If $I_C = I_{cz}$, then the mass dependence of dJ/dt would be far less steep with increasing stellar mass, as shown in Figure 7 (left panel, unfilled circles), and undefined blueward of $B-V = 0.45$, where both I_{cz} and $T(B-V)$ are zero. (The two curves coincide for very low mass fully convective stars.)

Expression (26) can be compared directly with the $\Omega > \Omega_{crit}$ part of the prior loss rate expression (7) which is:

$$\frac{dJ}{dt} = -K_w\sqrt{\frac{R}{M}}\Omega_{crit}^2\Omega. \tag{27}$$

We see that the difference between them boils down to the difference between $I_C/T(B-V)$ and $\sqrt{R/M}\Omega_{crit}^2$. The case of a constant Ω_{crit} is displayed graphically in Figure 7 (left panel, squares), normalized to the
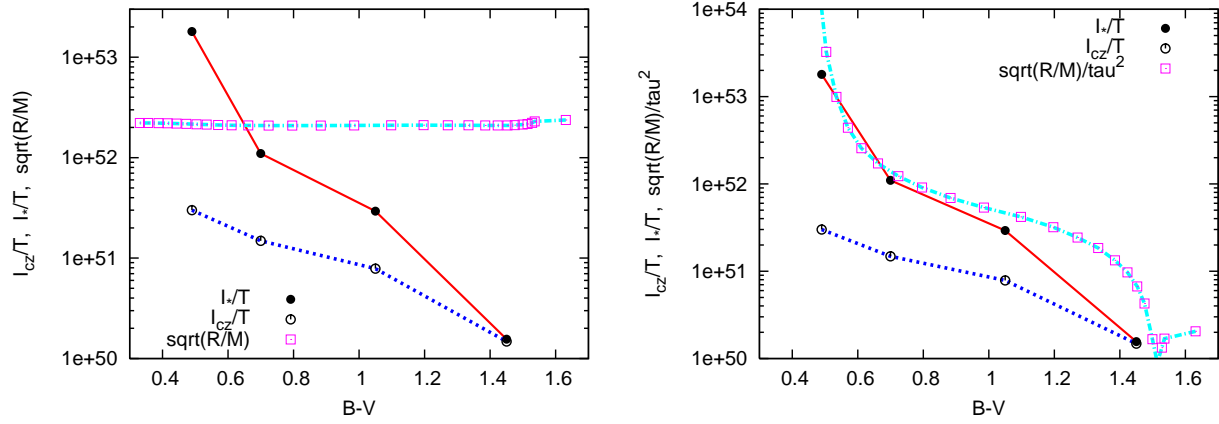
Fig. 7.— **Left:** $I_*/T(B-V)$ (filled circles) and $I_{cz}/T(B-V)$ (unfilled circles), both candidates for the empirical C sequence mass dependence, are compared with the prior $\sqrt{R/M}$ C sequence mass dependence (squares). While the first two decline steadily with increasing $B-V$ color, $\sqrt{R/M}$ is flat. **Right:** The same functions I_*/T and I_{cz}/T are compared with the Krishnamurthi et al. (1997) proposal that $\Omega_{\rm crit} \propto 1/\tau_c$ (instead of constant as at left). We see that the last is similar to $I_*/T(B-V)$ and $I_{cz}/T(B-V)$, but more complicated.

solar mass model with $I_C = I_*$. We note that, as expected, it is a remarkably flat function of stellar mass, suggesting quite different expectations for the corresponding angular momentum loss rate than for the $I_C = I_{cz}$ and $I_C = I_*$ cases discussed above. Because of various issues discussed in their paper, Krishnamurthi et al. (1997) decided to abandon a constant $\Omega_{\rm crit}$, and suggested a scaling where $\Omega_{\rm crit} \propto 1/\tau_c$. We display the result in Figure 7 (right panel, squares), normalized to the solar mass model with $I_C = I_*$. We find that this scaling gives results similar to the $I_C = I_*$ or $I_C = I_{cz}$ cases above, which are both conceptually simpler.

3.3. Combined expression

To summarize the conclusions of this section, combining the two expressions above, the empirical acceleration rate becomes

$$\frac{d\Omega}{dt} = -\begin{cases} \Omega/T(B-V) & \text{for the C sequence} \\ \Omega^3\, f^2(B-V)/8\pi^2 & \text{for the I sequence,} \end{cases} \tag{28}$$

where $T(B-V)$ and $f(B-V)$ are derivable from open cluster rotation period data. This leads immediately to the following expression for the angular momentum loss rate

$$\frac{dJ}{dt} = -\begin{cases} \Omega\left\{I_C/T(B-V) - dI_C/dt\right\} & \text{for the C sequence} \\ \Omega^3\, I_* f^2(B-V)/8\pi^2 & \text{for the I sequence,} \end{cases} \tag{29}$$

and on further assuming that I_C, the relevant moment of inertia on the C sequence is constant in time, we get

$$\frac{dJ}{dt} = -\begin{cases} \Omega\, I_C/T(B-V) & \text{for the C sequence} \\ \Omega^3\, I_* f^2(B-V)/8\pi^2 & \text{for the I sequence.} \end{cases} \tag{30}$$

4. Interpretation of $T(B-V)$ and $f(B-V)$

We now consider, in turn, possible interpretations for $T(B-V)$ and $f(B-V)$. We will suggest that both are related, albeit differently, to the convective turnover timescale in cool stars.

4.1. $T(B-V)$

We recall that $T(B-V)$ is defined by

$$P_C = P_0 e^{t/T(B-V)}, \tag{31}$$

where P_C represents the rotation periods of C sequence stars in open clusters, P_0 is the appropriate initial period, and t is the age. $T(B-V)$ is the appropriate mass-dependent timescale. Accordingly, $T(B-V)$ must have dimensions of time.

In Section 2.2, where it was first introduced, $T(B-V)$ was discussed in detail. Here, we merely reiterate that the observations currently define it in terms of four data points. It drops to zero at $B-V \sim 0.47$, and increases steadily for lower mass stars, as shown in Figure 3.

We are struck by the similarity between $T(B-V)$ and the convective turnover timescale, τ_c, in cool stars, which we have calculated and list in Table 1 as a function of stellar mass and $[UBVRIJHK]$ colors. τ_c and $T(B-V)$ are displayed and compared in Figure 8 (left) and Figure 8 (right) on linear and logarithmic scales respectively. In this comparison, and the next, we use the global turnover timescale. However, the (somewhat shorter) local timescale at the base of the convection zone is also provided in Table 1. More details about the calculation can be found in Kim & Demarque (1996).
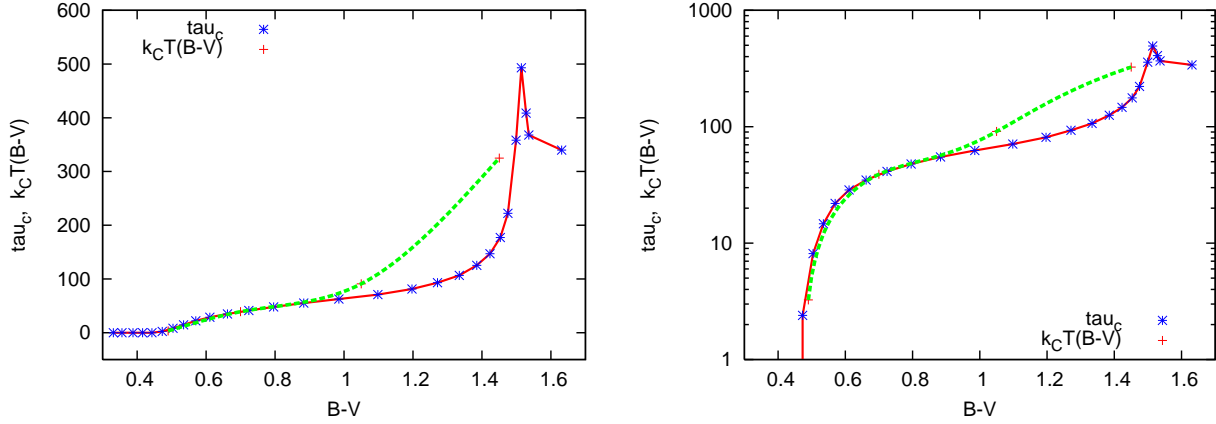
Fig. 8.— **Left:** τ_c (asterisks) and $k_C T(B-V)$ (pluses) are plotted against $B-V$ color on a linear scale. Normalization at solar mass requires $k_C = 0.65$d/Myr. A cubic spline fit to the $T(B-V)$ data points (dashed green line) is also displayed. **Right:** The same comparison on a logarithmic scale, emphasizing the blue region where both functions decline sharply. We see that τ_c and $k_C T(B-V)$ are in reasonable agreement.

The behaviors of τ_c and $T(B-V)$ suggest the identification

$$T(B-V) = \frac{\tau_c}{k_C}, \tag{32}$$

where τ_c is the convective turnover timescale, and k_C is a (dimensionless) constant. We find that $k_C \approx 0.65$ d/Myr. Indeed, with this choice of k_C, the two curves are in reasonable agreement. (At this early stage, it is not possible to comment on the detectability or importance of the slight non-monotonicity of τ_c for the lowest stellar masses.)

4.2. $f(B-V)$

The situation with $f(B-V)$ is similar to that with $T(B-V)$ above. Recall that $f(B-V)$ is defined by the expression

$$P_I = f(B-V) \times g(t), \tag{33}$$

where P_I represents the rotation periods of I sequence stars in open clusters, and $f(B-V)$ and $g(t)$ are separable functions, respectively, of the $B-V$ color and age, t, of the stars. To a precision adequate for the present purposes, $g(t) = \sqrt{t}$. Because P_I has the dimensions of *time* we infer that $f(B-V)$ must have the dimensions of $\sqrt{time}$, or that $f^2(B-V)$ must have the dimensions of *time*.

The rotation periods of the I sequence stars decrease sharply for stars bluer than the Sun, approaching zero for $B-V = 0.45$. For stars redder than the Sun, the rotation periods increase all the way till at least $0.5 M_\odot$. This behavior is again remarkably reminiscent of the behavior of the convective turnover timescale but dimensionally ought to be associated with its square root. We therefore propose the identification

$$f^2(B-V) = \frac{2\tau_c}{k_I}, \tag{34}$$

where k_I is a dimensionless constant, and τ_c is again the convective turnover timescale. The factor of 2 is added purely for later convenience. We find that $k_I \sim 740$ or 1340 Myr/d, where the smaller value is relevant to the $f(B-V)$ determination of Meibom et al. (2009) while the larger value is relevant to the determination of Mamajek & Hillenbrand (2008). The functions $2\tau_c$ and $k_I f^2(B-V)$ are compared graphically in Figure 9 and indeed they are in reasonable agreement.

4.3. Symmetric combination

The above interpretations of $T(B-V)$ and of $f(B-V)$ are appealingly symmetric. Although there are two timescales in the problem, captured by the dimensionless constants, k_C and k_I, the spin-down of both the C- and I sequences in open clusters appear to be connected by one underlying variable, τ_c, the convective turnover timescale in cool stars. As a result, it appears that the underlying variables in stellar rotation appear to be P, t, and τ_c, the last encapsulating the dependence on stellar mass.

Consequently, placing the identifications above into Equation (28), we suggest the following expression for the deceleration of cool stars:

$$\frac{d\Omega}{dt} = - \begin{cases} \Omega\,k_C/\tau_c & \text{for the C sequence} \\ \Omega^3\,\tau_c/4\pi^2 k_I & \text{for the I sequence} \end{cases} \tag{35}$$

or, on changing variable from Ω to $P = 2\pi/\Omega$, we get

$$\frac{dP}{dt} = \begin{cases} k_C P/\tau_c & \text{for the C sequence} \\ \tau_c/k_I P & \text{for the I sequence} \end{cases} \tag{36}$$

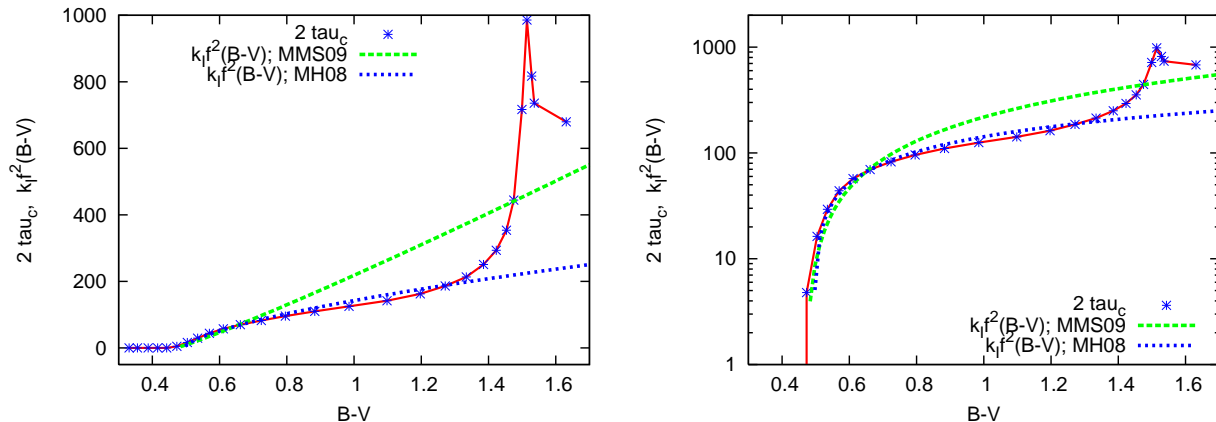which displays the symmetry of the problem explicitly.

Fig. 9.— **Left:** $2\tau_c$ (asterisks) and $k_I f^2(B-V)$ are plotted against $B-V$ color on a linear scale. The dashed green curve is the Meibom et al. (2009) determination of $f(B-V)$, while the dotted blue curve is that of Mamajek & Hillenbrand (2008). Normalization at solar mass requires $k_I = 740\,\mathrm{Myr/d}$ and $1340\,\mathrm{Myr/d}$ respectively. **Right:** The same comparison on a logarithmic scale, emphasizing the blue region where all the functions decline sharply. We see that $2\tau_c$ and $k_I f^2(B-V)$ are in reasonable agreement.

5. Relationship with stellar activity physics

In a classic paper, Noyes et al. (1984) proposed that the fractional chromospheric emission from cool stars was dependent, not just on the rotation period, P, but also on stellar mass. Inspired by theoretical work by Durney & Latour (1978), they identified the convective turnover timescale, $\tau_c(B-V)$, as the most likely variable to capture the dependence on stellar mass, and demonstrated that the fractional chromospheric emission of cool stars, R'_{HK}, was more tightly dependent on the Rossby Number, $Ro = P/\tau_c$, than on P alone. Soon thereafter, the transition-region ultraviolet emission and coronal X-Ray activity of cool stars were also shown to behave similarly (Vilhu 1984; Simon et al. 1985), and it is now generally accepted that stellar activity is associated more closely with the Rossby Number than with the rotation period alone.

This work brings the rotational evolution of cool stars under the same umbrella by showing that dP/dt is a function of P/τ_c. Consequently, the rotation rate of a cool star appears to be a deterministic function of its age and its mass, the latter appearing through the convective turnover timescale, τ_c.

6. Conclusions

We have shown here that open cluster rotation period data allow the empirical determination of an expression for angular momentum loss from cool stars on the main-sequence. The relationship has a bifurcation, as before, that corresponds to an observed bifurcation in the rotation periods of open cluster stars.

One component of the relationship, the dependence on rotation rate, persists from prior work. The remaining dependence is captured by two observationally determinable functions of the mass or color of a star, which we write as $f(B-V)$ and $T(B-V)$ for the two tines of the relationship, respectively. Transformations to mass or other desired colors in the set $[UBVRIJHK]$ can be accomplished using Table 1.

We derive an empirical expression for the angular momentum loss rate, which is

$$\frac{dJ}{dt} = - \begin{cases} \Omega\left\{{}^{I_C}/_T - {}^{dI_C}/_{dt}\right\} & \text{for the C sequence} \\ {}^{\Omega^3\, I_*\, f^2}/_{8\pi^2} & \text{for the I sequence} \end{cases} \tag{37}$$

and which simplifies to

$$\frac{dJ}{dt} = - \begin{cases} {}^{\Omega I_C}/_T & \text{for the C sequence} \\ {}^{\Omega^3\, I_*\, f^2}/_{8\pi^2} & \text{for the I sequence.} \end{cases} \tag{38}$$

if the relevant moment of inertia, I_C, on the C sequence, is not time-varying.

Both $f(B-V)$ and $T(B-V)$ appear to be related to the convective turnover timescale, τ_c in stars. We suggest the identifications

$$T(B-V) = \frac{\tau_c}{k_C} \tag{39}$$

and

$$f^2(B-V) = \frac{2\tau_c}{k_I}, \tag{40}$$

where k_C and k_I are two dimensionless constants appropriate to the C- and I sequences, and are respectively approximately $0.65\,\mathrm{d/Myr}$ and 740 or $1340\,\mathrm{Myr/d}$, depending on which of two particular forms one uses for $f(B-V)$.

Consequently our final expression for the angular momentum loss rate is

$$\frac{dJ}{dt} = - \begin{cases} \Omega\left\{{}^{k_C I_C}/_{\tau_c} - {}^{dI_C}/_{dt}\right\} & \text{for the C sequence} \\ {}^{\Omega^3\, I_*\, \tau_c}/_{4\pi^2 k_I} & \text{for the I sequence} \end{cases} \tag{41}$$

and that for the deceleration of cool stars is

$$\frac{dP}{dt} = \begin{cases} {}^{k_C P}/_{\tau_c} & \text{for the C sequence} \\ {}^{\tau_c}/_{k_I P} & \text{for the I sequence} \end{cases} \tag{42}$$

which is both symmetric and dimensionally correct. We see that in this model, the evolution of the rotation period, P, depends only on the age, t, and the convective turnover timescale, τ_c, which encodes the mass dependence of rotation.

Finally, we have pointed out that this model for the rotational evolution of stars makes a natural connection to stellar activity physics, where P/τ_c is the preferred independent variable.

These ideas are developed further in following papers into a simple nonlinear model for the rotational evolution of cool stars from C- to I-type, and a companion paper will provide more general grids of related calculated quantities on and off the main-sequence.

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Table 1. Calculated Properties of Solar Metallicity Main-sequence Stellar Models at 500 Myr, as used in the text

Mass $(M_\odot)$	log T (K)	$\log L/L_\odot$ (-)	Age (Gyr)	Global τ_c (d)	Local τ_c (d)	Moment of Inertia ($g\,cm^2$)				Lejeune et al. (1997, 1998) Colors								Green et al. (1987) Colors				
						Con. Core	Rad. Zone	Con. Env	Total	U	B	V	R	I	J	H	K	U	B	V	R	I
0.15	3.50961	-2.53866	0.500	3.398e+02	1.628e+02	0.000e+00	0.000e+00	8.922e+51	8.922e+51	16.391	15.056	13.425	12.221	10.701	9.254	8.640	8.385	16.110	14.982	13.420	12.265	10.819
0.20	3.52474	-2.28585	0.500	3.679e+02	1.784e+02	0.000e+00	0.000e+00	1.839e+52	1.839e+52	15.172	14.000	12.464	11.357	9.986	8.666	8.041	7.805	15.259	14.095	12.563	11.471	10.129
0.25	3.53505	-2.10378	0.500	4.086e+02	2.030e+02	0.000e+00	0.000e+00	3.166e+52	3.166e+52	14.472	13.316	11.788	10.719	9.484	8.282	7.652	7.432	14.575	13.434	11.939	10.896	9.634
0.30	3.54339	-1.95807	0.500	4.930e+02	2.565e+02	0.000e+00	0.000e+00	4.907e+52	4.907e+52	13.896	12.761	11.247	10.214	9.089	7.976	7.339	7.133	14.069	12.916	11.442	10.436	9.238
0.35	3.55082	-1.81901	0.500	3.584e+02	1.718e+02	1.330e+51	2.140e+51	6.991e+52	7.338e+52	13.457	12.308	10.809	9.802	8.729	7.641	7.006	6.806	13.596	12.440	10.986	10.004	8.864
0.40	3.55929	-1.69122	0.500	2.221e+02	1.133e+02	7.920e+50	2.080e+52	7.952e+52	1.011e+53	13.042	11.873	10.398	9.422	8.402	7.332	6.694	6.503	13.163	11.983	10.545	9.588	8.515
0.45	3.56950	-1.55867	0.500	1.769e+02	9.196e+01	7.784e+50	4.435e+52	8.833e+52	1.334e+53	12.600	11.402	9.949	9.008	8.056	7.014	6.377	6.195	12.653	11.470	10.061	9.135	8.148
0.50	3.58154	-1.42169	0.500	1.468e+02	7.728e+01	6.085e+50	7.457e+52	9.507e+52	1.702e+53	12.117	10.895	9.472	8.566	7.680	6.705	6.065	5.897	12.139	10.937	9.561	8.671	7.768
0.55	3.59551	-1.28094	0.500	1.254e+02	6.612e+01	3.030e+50	1.117e+53	9.931e+52	2.114e+53	11.582	10.343	8.958	8.086	7.270	6.406	5.769	5.617	11.577	10.370	9.039	8.188	7.376
0.60	3.61129	-1.13877	0.500	1.068e+02	5.729e+01	6.108e+49	1.517e+53	1.044e+53	2.562e+53	11.029	9.789	8.455	7.638	6.905	6.080	5.443	5.312	10.990	9.790	8.514	7.711	6.984
0.65	3.62846	-0.99758	0.500	9.305e+01	5.012e+01	0.000e+00	1.969e+53	1.070e+53	3.038e+53	10.439	9.230	7.959	7.217	6.580	5.736	5.113	5.002	10.374	9.204	7.995	7.245	6.597
0.70	3.64654	-0.85948	0.500	8.117e+01	4.400e+01	0.000e+00	2.454e+53	1.082e+53	3.536e+53	9.832	8.696	7.499	6.819	6.253	5.415	4.810	4.708	9.726	8.651	7.511	6.821	6.233
0.75	3.66575	-0.72461	0.500	7.089e+01	3.887e+01	0.000e+00	2.954e+53	1.090e+53	4.044e+53	9.124	8.120	7.022	6.420	5.926	5.123	4.555	4.459	9.054	8.117	7.052	6.422	5.886
0.80	3.68619	-0.59368	0.500	6.254e+01	3.458e+01	0.000e+00	3.460e+53	1.093e+53	4.553e+53	8.349	7.543	6.558	6.032	5.594	4.866	4.346	4.258	8.345	7.585	6.603	6.039	5.553
0.85	3.70568	-0.46899	0.500	5.501e+01	3.079e+01	0.000e+00	3.994e+53	1.081e+53	5.074e+53	7.634	7.040	6.157	5.681	5.270	4.632	4.153	4.074	7.686	7.091	6.190	5.684	5.243
0.90	3.72368	-0.35030	0.500	4.790e+01	2.729e+01	0.000e+00	4.572e+53	1.036e+53	5.608e+53	7.015	6.605	5.809	5.367	4.974	4.404	3.959	3.886	7.089	6.644	5.822	5.366	4.958
0.95	3.73984	-0.23755	0.500	4.124e+01	2.400e+01	0.000e+00	5.191e+53	9.679e+52	6.159e+53	6.475	6.204	5.480	5.070	4.695	4.191	3.790	3.723	6.536	6.226	5.477	5.065	4.689
1.00	3.75439	-0.12957	0.500	3.487e+01	2.080e+01	0.000e+00	5.852e+53	8.648e+52	6.717e+53	5.992	5.835	5.173	4.797	4.446	3.987	3.625	3.568	6.081	5.859	5.175	4.795	4.444
1.05	3.76739	-0.02597	0.500	2.864e+01	1.757e+01	0.000e+00	6.570e+53	7.092e+52	7.279e+53	5.583	5.499	4.888	4.541	4.213	3.779	3.446	3.394	5.659	5.515	4.888	4.538	4.209
1.10	3.77903	0.07324	0.500	2.193e+01	1.419e+01	4.481e+50	7.310e+53	5.254e+52	7.840e+53	5.226	5.190	4.620	4.298	3.992	3.573	3.263	3.212	5.264	5.186	4.615	4.291	3.983
1.15	3.78948	0.16746	0.500	1.467e+01	1.069e+01	1.960e+51	8.049e+53	3.331e+52	8.401e+53	4.911	4.898	4.363	4.061	3.774	3.392	3.107	3.060	4.951	4.901	4.369	4.066	3.774
1.20	3.79899	0.25725	0.500	8.141e+00	7.440e+00	3.944e+51	8.770e+53	1.525e+52	8.962e+53	4.617	4.623	4.119	3.837	3.566	3.218	2.959	2.916	4.652	4.627	4.134	3.849	3.573
1.25	3.80807	0.34281	0.500	2.394e+00	0.000e+00	5.794e+51	9.448e+53	2.806e+51	9.534e+53	4.344	4.363	3.890	3.625	3.371	3.056	2.822	2.782	4.370	4.368	3.911	3.644	3.383
1.30	3.81760	0.42605	0.500	0.000e+00	0.000e+00	6.669e+51	1.004e+54	0.000e+00	1.010e+54	4.084	4.110	3.667	3.420	3.182	2.902	2.694	2.657	4.110	4.115	3.693	3.445	3.200
1.35	3.82645	0.50530	0.500	0.000e+00	0.000e+00	7.989e+51	1.065e+54	0.000e+00	1.073e+54	3.848	3.873	3.457	3.226	3.001	2.754	2.570	2.535	3.884	3.884	3.490	3.257	3.027
1.40	3.83525	0.58081	0.500	0.000e+00	0.000e+00	9.578e+51	1.132e+54	0.000e+00	1.141e+54	3.623	3.648	3.260	3.048	2.838	2.613	2.448	2.415	3.668	3.659	3.294	3.077	2.862
1.45	3.84458	0.65226	0.500	0.000e+00	0.000e+00	1.106e+52	1.206e+54	0.000e+00	1.218e+54	3.404	3.432	3.076	2.885	2.693	2.483	2.337	2.306	3.461	3.446	3.109	2.910	2.711
1.50	3.85476	0.72048	0.500	0.000e+00	0.000e+00	1.331e+52	1.288e+54	0.000e+00	1.302e+54	3.227	3.227	2.896	2.721	2.539	2.366	2.232	2.201	3.278	3.242	2.934	2.752	2.569